WTS TUTORING

WTS

CALCULUS

GRADE : 12

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: WTS TUTORS

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FACEBOOK P. : WTS MATHS & SCEINCE TUTORING

WHERE TO START MATHS & SCIENCE TUTORING			
FINAL EXAMINATION			
GRADE	12		
SUBJECT	MATHEMATICS		
PAPER	PAPER 1		
DURATION OF THE PAPER	3 HOURS		
TOTAL MARKS	150		
NUMBER OF QUESTIONS	10 – 12		
QUESTION PAPER FORMAT	LEVEL 1 questions	Knowledge	20%
	LEVEL 2 questions	Routine procedures	35%
	LEVEL 3 questions	Complex procedures	30%
	LEVEL 4 questions	Problem solving	15%
EXPECTED WORK COVERAGE			
ALGEBRA, EQUATIONS AND INEQUALITIES	25±3 marks		
NUMBER PATTERNS	25±3 marks		
FINANCE, GROWTH AND DECAY	15±3 marks		
FUNCTIONS AND GRAPHS	35±3 marks		
DIFFERENTIAL CALCULUS	35±3 marks		
PROBABILITY	15±3 marks		

➢ ALGEBRA

Polynomial:……………………………………………… is a polynomial of degree…………

A linear Polynomial:…………………………………….. is a polynomial of degree…………

A quadratic Polynomial:………………………………….. is a polynomial of degree………...

A cubic Polynomial:………………………………………. is a polynomial of degree………..

➢ Factorising a cubic polynomial

A. Sum and difference of two cubes

- Formula for sum : $(x^3 + a^3) = (x + a)((x^2 - ax + a^2)$
- Formula for difference : $(x^3 - a^3) = (x - a)((x^2 + ax + a^2)$

Kwv 1

Factorise each of the following:

a) $8x^3 - 64$

b) $x^3 + 27$

c) $x^3 - 8$

d) $x^3 + 1$

B. Grouping in pairs

- It is a factorising method that can be used when an expression has four or more terms and then therefore terms can be grouped in pairs
- Positive sign must be in between the brackets

Kwv 1

Factorise each of the following

a) $4x^3 + 8x^2 + 3x + 6$
b) $6x^3 + 3x^2 - 12x - 4$

C. Solving cubic using synthetic method

- there must be four terms and if one term is missing you must use zero instead
- ensure you take a coefficient with the sign

Kwv 1

a) divide $x^3 - x^2 + 4x - 3 \; by \; x + 2$

b) divide $x^3 - 12x + 16 \; by \; x - 2$

i. The remainder theorem

- If a polynomial $f(x)$ is divided by a linear polynomial $ax - b$, then the remainder is $f(\frac{b}{a})$
- Firstly substitute and equate to the remainder

ii. The factor theorem

- If $f(x)$ is a polynomial such as that $f\left(\frac{b}{a}\right) = 0$, then $ax - b$ is a factor of $f(x)$
- In factor theorem the remainder is zero

Kwv 1

1. If $(x - 3)$ is a factor of $h(x) = 4x^3 - (a + 16)x + 24 - 34$

a) Determine the value(s) of a.

b) Hence factorise $h(x)$ completely for the value(s) of a determined in a

2 If $f(x) = x^3 + mx^2 + nx + c$

a) If $(x - 1)$ is a factor of $f(x)$ and $f(x)$ leaves a remainder of 8 when divided by $(x + 1)$ Calculate the values of m and n.

b) Factorise $f(x)$ completely.

3) $f(x) = 2x^3 + x^2 - ax + b$ Is exactly divided by $2x - 1$, but leaves the remainder of 6 when divided by$(x + 1)$. Find the value of a and b.

4) When $x^3 + mx^2 + nx + 1$ is divided by $x - 2$ the remainder is 9, when divided by $x - 2$ the remainder is 19. Find the values of m and n.

5) If $(2x - 1)$ is a factor of $px^3 - 3x^2 - 3x + p$, determine the values of p.

6) Given: $f(x) = 2x^3 - 3x^2 - 11x + 6$

a) Prove that $(x + 2)$ is a factor of $f(x)$.
b) Hence factorise $f(x)$ completely.
c) Now determine the values of x if $f(x) = 0$.
d) Draw a sketch graph of f without indicating the coordinates of the turning point. Only indicate the intercept with the axes.

7 Using the x remainder theorem:
a) Show that $(x + 2)$ is a factor of $f(x) = x^3 + x^2 - 13x - 22$.
b) Hence factorise $f(x)$ completely.

D. Solving cubic equations

- The standard form is**:** $f(x) = ax^3 + bx^2 + cx + c$
- Let $f(x) = 0$
- And you can use synthetic or inspection {smile method}

Kwv 1

Solve for x

a) $(x - 3)(x + 2)(2x + 5) = 0$
b) $x^4 + 2x^3 - 4x^2 = 0$

c) $1 - x^3 = 0$

d) $6x^2 - x^3 = 0$

e) $x^3 - 4x^2 - 11x + 30 = 0$

f) $x^3 - 12x - 16 = 0$

g) $(x - 1)(x + 3)^2 = 0$

➢ CALULUS

A. Average gradient

The average gradient between two points is the gradient of a straight line drawn between the two points.

Kwv 1

Consider: $\boldsymbol{f(x) = -x^2 - 3x + 1}$

1. What is the average gradient between $x = 1$ and $x = -2$.

2. What is the rate of change of f between the $(2:3)$ and $(4:15)$.

B. Gradient at a single point on a curve

- derivative is the gradient
- $f'(a) = m \quad \{a \text{ is the value of } x \text{ at that point}\}$

Kwv 1

Consider: $f(\boldsymbol{x}) = \boldsymbol{-x^2 - 3x + 1}$

1. Determine the gradient at $x = 1$
2. Determine the gradient at $x = -2$
3. Determine the point of contact for $x = 1$
4. Determine the point of contact for $x = -2$

Kwv 2

Consider: $g(x) = -x^2 - 3x + 1$, and then determine the following:

a) $g(3)$
b) $g(x + 3)$
c) $g(-x)$
d) $g(h)$
e) $g(x + h)$
f) $g'(3)$

C. Finding the derivative

- **Finding the derivative from first principles**

The derivative of a function $f(x)$ is written as $f'(x)$ and is defined by:-----------------------

STEPS:

- make it a point that you copy the formula as it is in the formula sheet
- simply substitute $(x + h)$ where there is x
- where there is a fraction you must find the LCD

Kwv 1

Calculate the derivative of the following from first principles:

a. $f(x) = 5$

b. $f(x) = 5x$

c. $f(x) = -5x^2$

d. $f(x) = -5x^2 + k$

e. $f(x) = 5 - 5x^2$

a. $f(x) = -5x^2 - 3$

b. $f(x) = -5x^2 - 3x$

c. $f(x) = -5x^2 + 3x + 2$

d. $f(x) = -5x^3$

e. $f(x) = -\frac{2}{x}$

f. $f(x) = -\frac{2}{5x}$

g. $xy = 5$

h. $f(x) = \frac{3}{x} + 2$

i. $f(x) = \frac{1}{3}x^2 - 1$

j. $f(x) = \sqrt{x}$

Kwv 2

Given: $f(x) = 4 - 3x^2$

i. Determine $f'(x)$ from the first principle

ii. A$(x: -23)$, where $x > 0$ at B $(-2: y)$ are points on the graph of f.

Calculate the numerical value of average gradient of f between A and B.

Kwv 3

Sakhile determines $f'(b)$, the derivative of a certain function f at $x = b$ and arrives at the answer $\lim_{h \to 0} \frac{\sqrt{4+h}-2}{h}$, write down the equation of *f* and the value of *b.*

D. Rules of derivative

- **Finding the derivative using the rule for differentiating**

Before you differentiate you might need to:

- Expand brackets because you have no rule for differentiate product.
- Rewrite terms which are square roots, cube roots, other roots as exponentials so that you can use the rule.
- Rewrite terms which are fractions , so that you can use the power rule
- Take note of notation we use for the derivative

Kwv 1

Write down all notations you know and discuss with your partner for its implication

Kwv 2

Write down five keys of power rules

Kwv 3

Determine the $g'(x)$ of the following:

a) $g(x) = \frac{2}{x^2}$
b) $g(x) = \frac{2}{3x^2}$
c) $g(x) = -\frac{x}{6}$
d) $g(x) = -\frac{2}{\sqrt{x}}$
e) $g(x) = \frac{2}{\sqrt[4]{x^3}}$
f) $g(x) = \frac{x^4-x^2+x+1}{x}$
g) $g(x) = \frac{x^2-x-6}{(x-3)}$
h) $g(x) = (3-x)^2$

Kwv 4

Determine each of the following:

a. Calculate $D_x\left[4 - \frac{4}{x^3} - \frac{1}{x^4}\right]$

b. $f(x) = \frac{4x^4+3x^2-x+2}{x}$; $f'(x)$

c. $f'(x)$ if $f(x) = \frac{x^2-x-6}{x-3}$

d. $f'(x)$ if $f(x) = \frac{x^3-5x^2+4x}{x-4}$

e. $g(x) = \frac{x^3-8}{x-2}$; $g'(x)$

f. $g(x) = \frac{x^3-8}{-x+2}$; $g'(x)$

g. $\frac{dy}{dx}$ if $y = \frac{2x^2-1}{\sqrt{x}}$

h. $\frac{dy}{dx}$ if $y = 3x^2.\sqrt[3]{8x^4}$

i. $f(x) = \left(x^2 - \sqrt{x}\right)^2;\ f'(x)$

j. $\frac{dy}{dx}$ if $y = \sqrt[3]{8x^{18}} - \frac{3}{4x^2}$

k. $\frac{dy}{dx}$ if $y = \left(\frac{x}{3} + \frac{3}{x}\right)^2$

l. $\frac{d}{dx}\left[\frac{\sqrt{x^3} - 5x + 2}{\sqrt{x}}\right]$

m. $xy + y = x^2 - 1;\quad \frac{dy}{dx}$

n. $8x^3 - 2xy + y = 1;\ \frac{dy}{dx}$

o. $\frac{d}{dx}[(3x^2) - 2\sqrt{x} + \frac{1}{2x}]$

p. $y^3 = (x^2 - 2)^3$

q. If $y = \frac{8}{x^3}$ and $z = \frac{y^2 - 1}{y}$, determine:

i. $\frac{dy}{dx}$

ii. $\frac{dx}{dy}$

iii. $\frac{dz}{dy}$

iv. $\frac{dz}{dx}$

r. $\boldsymbol{\frac{dy}{dx}\ if\ y = \sqrt{x^3} - \frac{5}{x} + \frac{1}{2}a}$

Kwv 5

a) Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up.

E. Conclusion

Uses of the derivative:

- To find the equation of a tangent line
- To locate stationary points
- To find where a maximum or minimum value occur
- To describe rates of change
- To draw cubic polynomials

F. Tangent Equation

a) Finding the equation of a tangent line

The slope of the tangent line to the graph at a point is equal to the derivative of the function at that point. So to find the equation of the tangent line to $f(x)$ at $x = a$

- Take the derivative, and then
- Evaluate the derivative at $x = a$ i.e. to calculate $f'(a) = m$ to get the gradient of the tangent line,
- calculate the y-value at $x = a$ i.e. calculate $f(a)$ { point of contact}
- and lastly use the equation of the line:$y = mx + c \quad or \quad y - y = m(x - x)$

b) Finding the unknown variables

- For two variables you need two points and hence equations
- For three variables you need to work with the first two variables and use one point for remaining variable
- For eq 1: $f'(a) = m$
- For eq 2: substitute the points of contact and if not given simple calculate $f(a)$
- Solve them simultaneously

Kwv 1

1. Given: $g(x) = x^3 + 4x^2 + 8x$

a. Determine $g(-2)$.

b. Determine $g'(-2)$.

c. Determine the equation of the tangent to g at $x = -2$ in the form $y = mx + c$.

d. Calculate the coordinates of the point of inflection of g

e. Show that g is increasing for all real value(s) of x.

Kwv 2

Determine the equation of the tangent to the curve of $f(x) = x^3 - 2x + 5$ at the point on the curve when $x = -2$.

Kwv 3

The tangent to the curve of $g(x) = 2x^3 + px^2 + px - 7$ has the equation $y = 5x - 8$.

a. Show that $(1; -3)$ is the point of contact of the tangent to the graph.

b. Hence, or otherwise, calculate the value of p and q.

Kwv 4

The curve with equation$y = ax^3 + bx + 4$ has a gradient of -4 at the point (1:8) on the curve. Determine the values of a and b.

Kwv 5

The equation of a tangent to the curve of $f(x) = ax^3 + bx$ and $y - x - 4 = 0$.

If the point of contact is $(-1; 3)$. Calculate the values of *a* and *b*

Kwv 6

The tangent to the curve of $y = x^2 - 4x$ is perpendicular to the line $y = -\frac{1}{2}x + 4$.

Find the equation of the tangent.

Kwv 7

Consider $f(x) = 2x^3 - 23x^2 + 80x - 84$

Determine the y-intercept of the tangents to *f* that has a slope of 40 (at where x is an integer)

Kwv 8

Given: $h(x) = ax^2, x > 0$.

Determine the value of a if it is given that $h^{-1}(8) = h^{-1}(4)$.

➢ CUBIC FUNCTION

- **Finding the stationary points of a function**

✓ When we are drawing the graph or looking for the max. or min. values of a function:
✓ it is useful to identify the turning points; these points are where the gradient of the function is zero. We solve $f'(x) = 0$ and substitute the x values into original equation for the y values.

- **Stationary points on a cubic function**

There are 3 stationary points

✓ Local maximum
✓ Local minimum
✓ Inflection point

Take note:

Because solving $f'(x) = 0$ can help us identify local max. or min. Points, we often use the derivative in solving an applied problem where we need to find a max. or min. value.

- **Sketching cubic function**

a) Shape of the graph:

- $a > 0$: increase, decrease and increase
- $a < 0$: decrease, increase and decrease

b) Find intercepts:

✓ for the y-intercept by finding $f(0)$
✓ for the x-intercepts by finding where $f(x) = 0$

Note:

✓ We first need to identify one factor using the factor theorem.
✓ The factor theorem says if $f(a) = 0$ and then $x - a$ is a factor of $f(x)$

c) For the x-value of the turning point:

- ✓ make $f'(x) = 0$
- ✓ solve for x
- ✓ For the y-value of the turning point substitute the x-value of turning point into the original equation.

d) For the x-value of the point of inflection make:

- ✓ $f''(x) = 0$ *and solve*
- ✓ Average the x-value of turning points (midpoint formula)
- ✓ Average the x-intercepts of a curve

e) For the y-value:

- ✓ substitute the x-value to the original
- ✓ y-midpoint formula for turning points

NB: Show that the concavity of changes at $x = a$

For: $a > 0$

- ➢ The graph changes from concave down to concave up at $x = a$

For: $a < 0$

- ➢ The graph changes from concave up to concave down at $x = a$

Note:

- ✓ Use the number line to calculate the concavity
- ✓ $f'' > 0$: concave up
- ✓ $f'' < 0$: concave down

f) Draw a neat sketch

Follow these steps:

- ✓ Indicate the axes, both x and y intercepts
- ✓ Indicate the turning points
- ✓ Consider the shape with max and min points

➢ **Reading from the graph**

✓ **For which value(s) of x will:**

a) $f'(x) > 0$ {where the graph is increasing}

b) $f'(x) < 0$ {where the graph is decreasing}

c) $f(x).g(x) > 0$ {where both graphs are above or below}

d) $f(x).g(x) < 0$ {One graph above and other one below the horizontal}

e) $f(x) > 0$ {above the horizontal}

f) $f(x) < 0$ {below the horizontal}

g) $g'(x) = 0$ {x-value of a turning point}

h) $g'(x) > 0$ *and* $f'(x) > 0$ {both graphs increasing}

i) $g'(x) < 0$ *and* $f'(x) < 0$ {both graphs decreasing}

j) $g(x) - f(x) = 1$ {graph $g(x)$ is above $f(x)$ and the distance is 1 unit}

k) $x.f(x) > 0$ {+ve x-value and +ve y-value or –ve x-value and –ve y-value}

l) $f(x).f'(x) < 0$ {+ve y-value and the decreasing curve}

Key note:

- Remember to manipulate the new given equation into the original one
- Maximum and minimum values means the y values of the turning point

- ✓ **Reading y-value**

For which value(s) of k will f(x) has:

- Only one real roots { horizontal line cuts the graph once}
- equal roots { horizontal line touches the turning points}
- Three distinct roots { horizontal line cuts the graph three times}

Note:

The line y = k + c has to intersect the graph f(x) at different places. Make sure the given equation is derived to the original equation and make y the subject of the formula

- ✓ **The transformation of cubic graph**

a) Translation

It affects the turning points

1. Horizontal translation: f (x + c)

Then x-value(s) of the turning point is translated c unit left (-ve) or right (+ve)

2. Vertical translation: f (x) + q

Then y-value of turning point will be translated q unit up (+ve) or down (-ve)

b) The reflection

It can affect the whole equation and the turning points. The reflection about axes

The reflections are as follows:

- reflected across x-axis $p(x) = -p(x)$
- reflected across y-axis $p(x) = p(-x)$
- reflected about the line $y = x$

c) The enlargement

- You simple multiply by the scale factor given
- Especial applies on the turning points

A CUBIC FUCTIONS

Kwv 1

1. P is the function defined by:

(i) $p_1 = x^3 - 4x^2 - 11x + 30$

(ii) $p_2 = -x^3 - x^2 + x + 10$

(iii) $p_3 = (x - 6)(x - 3)(x + 2)$

(iv) $p_4 = x^3 - 12x - 16$

Determine the following of each function:

a. Write down coordinates of $p(x) = 0$ and $p(0)$ or intercepts with the axes.

b. Calculate the coordinates of the turning point

c. Hence, sketch the graph of *p*.

d. Find the coordinate of inflection point/ point at which $f'(x)$ is a maximum

e. For which values of will;

i. $p(x) > 0$

ii. $p(x) < 0$

iii. $p'(x) > 0$

iv. $p'(x) < 0$

v. $x.p'(x) < 0$

vi. $x.p'(x) > 0$

vii. $p(x).p'(x) < 0$

viii. $p(x).p'(x) > 0$

f. For which value(s) of x the concavity of the graph will:

i. Concave up?

ii. Concave down?

g. Use the graph to determine the values of x for which the equation:

i. $p(x) = k$, have one real root, equal roots and 3 distinct roots.

ii. $p(x) - k = 0$, have one real root, equal roots and 3 distinct roots.

iii. $-x^3 - x^2 + x = k$, have one real root, equal roots and 3 distinct roots.

iv. $x^3 - 4x^2 - 11x = k$, have one real root, equal roots and 3 distinct roots.

v. Determine the value(s) of k for which $p = k$ has negative roots only

h. Calculate the average gradient between the turning points.

i. Determine the equation of the tangent to p at $x = 4$.

j. Write down the coordinate of turning point of:

i. $k = p(x) - 1$

ii. $k = p(x - 2) - 1$

iii. $k = -p(x)$

iv. $k = p(-x)$

k. Write down the coordinates of a turning points and equations of h if h is defined by

(i) $h(x) = 2p(x) - 4.$

(ii) Reflected across x-axis / $p(x) = -p(x)$

(iii) Reflected across y-axis / $p(x) = p(-x)$

(iv) Reflected about the line $y = x$

(v) Reflected about the line $y = -x$

Kwv 2

Given: $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

a. Show that f has a point of inflection at x = 2

b. sketch the graph of f, clearly indicating the intercepts with the axes and the turning points.

c. For which values of x will $y = -f$(x) be concave down?

d. use your graph to answer the following questions:

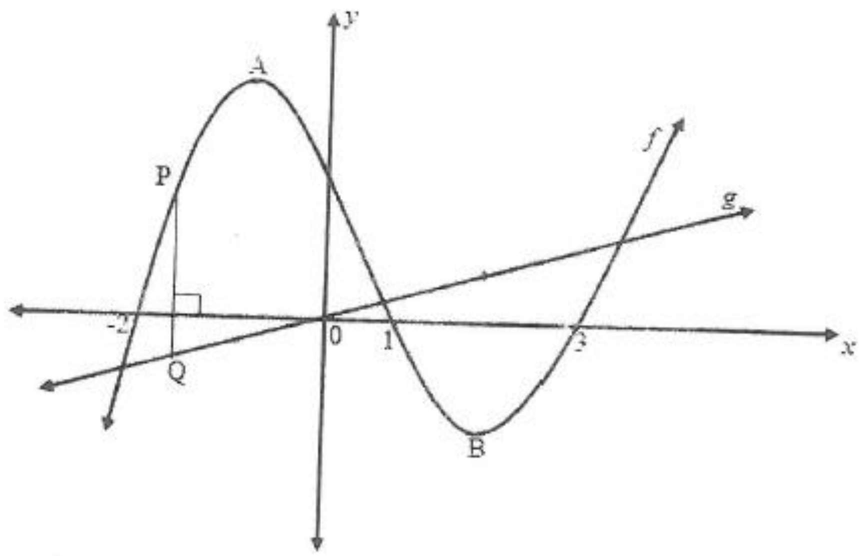
i. determine the coordinates of the local maximum of h if $h(x) = f(x-2) + 3$.

ii. May claims that $f'(2) = 1$ do you agree with May? Justify your answer.

Kwv 3

Given: $f(x) = x^3 + bx^2 + cx + d$ and $(x) = 2x$,

The graph of f intersects the x-axis at x = -2 ; x = 1 and x = 3 . The turning points of f are at A points B respectively, where $x_B > x_A$. Line PQ is perpendicular to the x-axis, with point P on f and point Q on g.



a. Show that the equation of f can be given as $f(x) = x^3 - 2x^2 - 5x + 6$

b. Calculate the coordinates of points A and B

c. Calculate the maximum length of line PQ , for the interval $-2 < x < 3$

d. The graph of f is concave down for x < k , calculate the value(s) of k.

e. Determine the equation to f at the point of inflection in the form $y = mx + c/$

B. EQUATION OF CUBIC FUNCTIONS

- ✓ **Finding the value of a, b, c, and d**

i) 3- x-intercepts given, if $\boldsymbol{the\ value\ of\ a\ is\ given}$

- ✓ Simple use: $y = a(x - x)(x - x)(x - x)$

ii) 3-x-intercepts and 1 point along the curve

- ✓ Simple use: $y = a(x - x)(x - x)(x - x)$

iii) Given only the turning point and or point

- ✓ derive $f(x) = 0$ and then substitute for two x –values of tuning point or
- ✓ substitute **x and y** direct to the equation $f(x)$

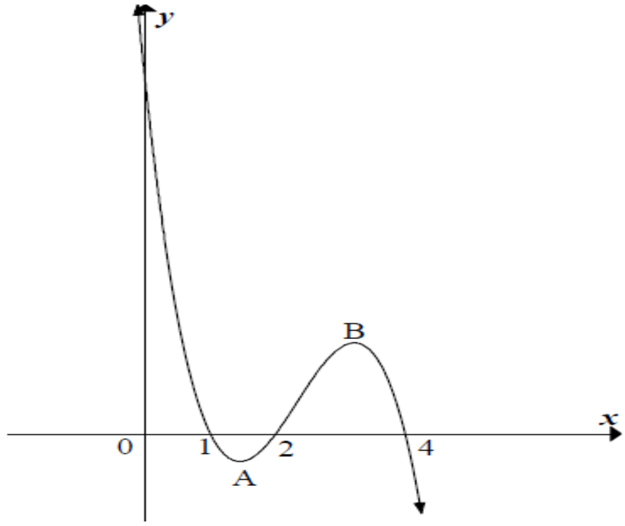
iv) Given the gradient of the tangent and one point

- ✓ $f'(x) = m$ and substitute given value of x { if not given equate the two equations}
- ✓ substitute x-value and y-value in $f(x)$and then solve simultaneously

Take note: Given y-intercept, automatically you have c-value or d-value

Kwv 1

The graph of $f(x) = -x^3 + ax^2 + bx + c$ is sketched below. The x-intercepts are indicated.

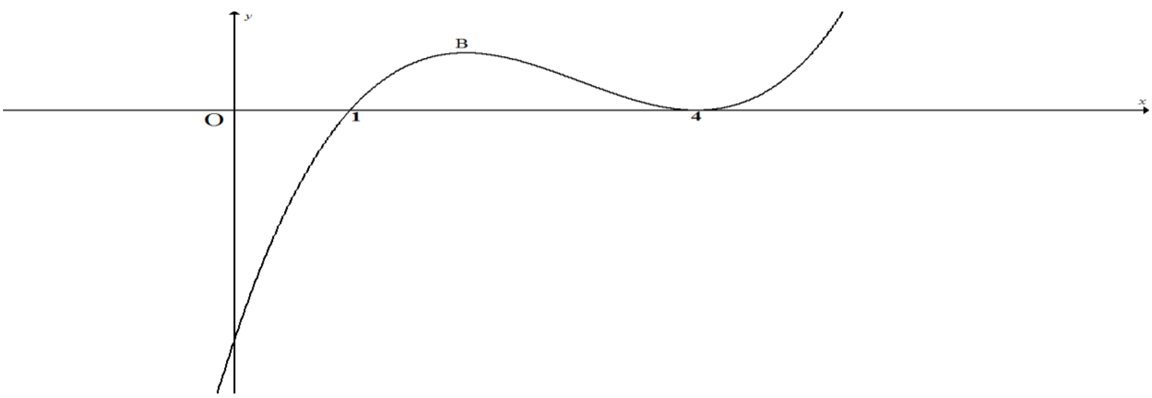


a. Calculate the values of a, b and c.

Kwv 2

The graph of a cubic function with equation $f(x) = x^3 + ax^2 + bx + c$ is drawn.

- $f(1) = f(4) = 0$
- f has a local maximum at B and a local minimum at $x = 4$.



a. Show that $a = -9$, $b = 24$ and $c = -16$.

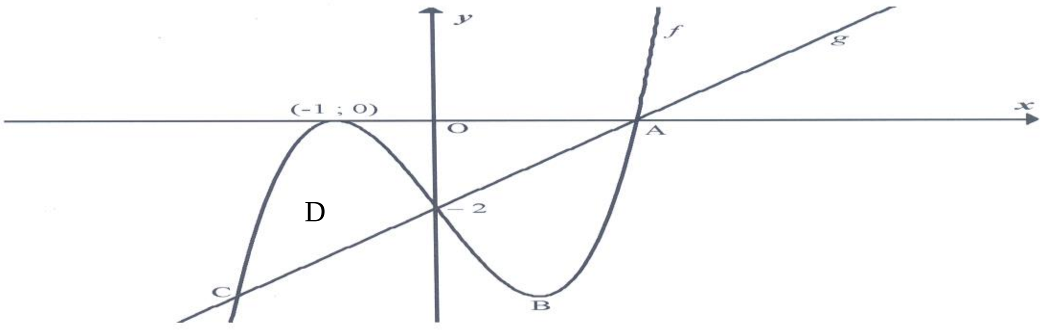
b. Calculate the x-coordinate of the point at which is a maximum.

c. Determine the value of x for which f is strictly increasing.

Kwv 3

The graph below represents the functions *f* and *g* with. $f(x) = ax^3 - cx - 2$ and

$g(x) = x - 2$. A and D(– 1; 0) are the *x*-intercepts of *f*. The graphs of *f* and *g* intersect at A and C.



a. Determine the coordinates of A.

b. Show by calculation that $a = 1$ and $c = 3$.

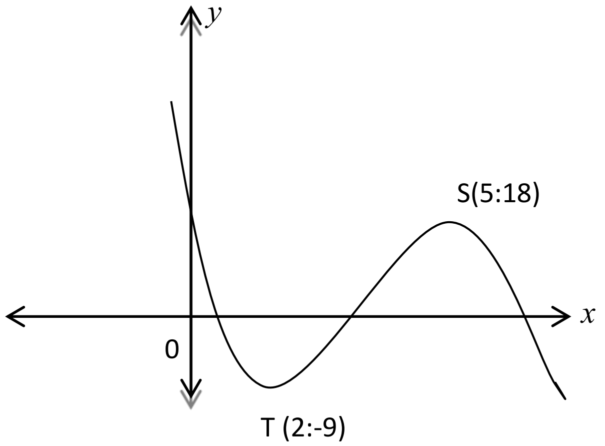
c. Calculate the coordinates of C

d. Calculate the average gradient between B and D

Kwv 4

The function $f(x) = -2x^3 + ax^2 + bx + c$ is sketched below.

The turning points of the graph of *f* are T (2:-9) and S (5:18).



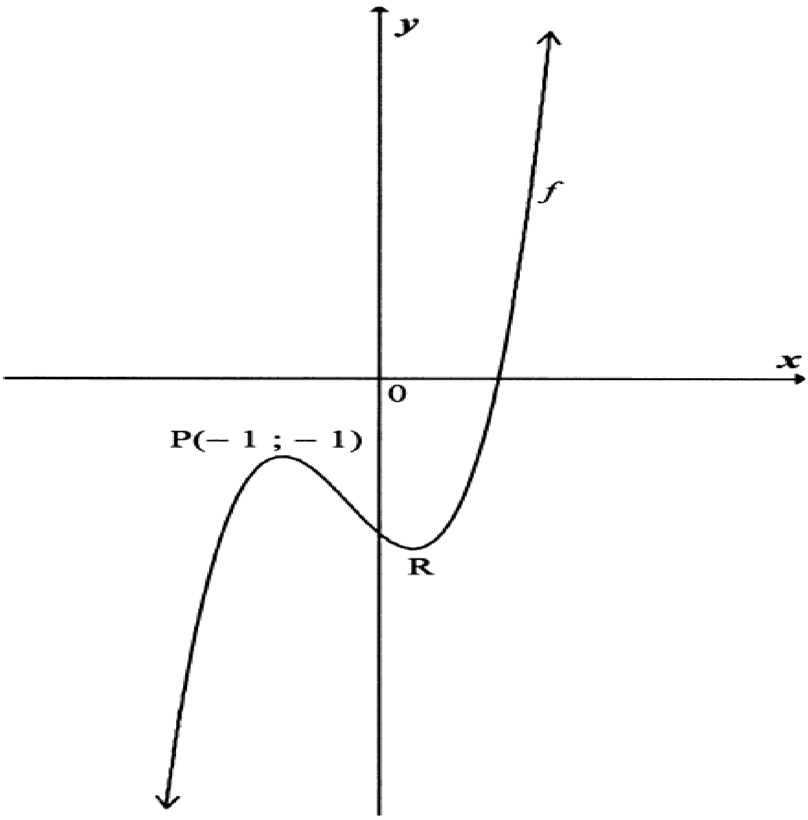
a. Show that $a = 21$, $b = -60$ and $c = 43$

b. Write down the coordinates of the turning points of $h(x) = f(x) - 3$

c. Write down the coordinates of the turning points of $h(x) = f(x - 3) - 3$

Kwv 5

The function defined by $f(x) = x^3 + ax^2 + bx - 2$ is sketched below.
P(− 1 ; − 1) and R are the turning points of f.



a) Show that $a = 1$ and $b = -1$.

b) Hence or otherwise, determine the x coordinate of R

c) Write down the coordinates of the turning points of h if h is defined by

$h(x) = 2f(x) - 4$

C. DERIVED GRAPH

- ✓ Parabola is a derivative of cubic graph
- ✓ Straight is the second derivative of cubic graph

- **Given y = f'(x) for parabola**

You must be able to find the following:

1. The equation of f'(x)

- ✓ if 3 points given: simple use $y = a(x - x)(x - x)$
- ✓ if turning point and one point given: simple use: $y = a(x - p)^2 + q$

2. The equation of f(x)

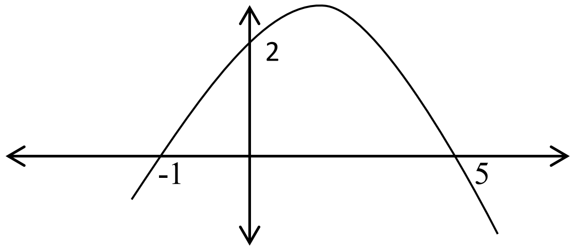
- ✓ you first need to derivative f(x)
- ✓ And then equate a with the value of a for both equation, and for b and c also (equating co-efficient)
- ✓ lastly write the final equation

3. The stationary points

- ✓ average the x-intercepts of the graph or use the midpoint formula
- ✓ x-intercept of the parabola give us the turning point since f'(x) = 0

Kwv 1

The sketch represents the curve of $y = f'(x)$ with$f'(x) = ax^3 + bx^2 + cx$.



a. What is the slope of the tangent to f at the point where $x = 0$?

b. Give the x-intercept of the curve f'.

c. Show that $x = \frac{-b}{3a}$ is the x-coordinate of the inflection point of f.

d. For which values of x is f decreasing?

e. For which values of x is $f'(x) > 0$.

e. Write down the value(s) of x that give local maximum and local minimum.

f. Hence, sketch the graph of $f(x)$.

g. Determine the equation of $f'(x)$.

Kwv 2

For the a certain function f, the first derivative is given as $f'(x) = 3x^2 + 8x - 3$.

a. calculate the x-coordinates of the stationary points of f.

b. for which values of x is f concave down?

c. determine the values of x for which is strictly increasing.

d. if it is further given that $f(x) = ax^3 + bx^2 + cx + d$ and $f(0) = -18$, determine the equation of f

D.PROPERTIES GIVEN

Given properties

- If there is prime that indicate the turning points
- If there is no prime that indicate the axes
- $f'(x) > 0$: indicate the increasing curve:
- $f'(x) < 0$: indicate the decreasing curve:

Kwv 1

The following information about a cubic polynomial, $y = f(x)$ is given:

- $f(-1) = 0$
- $f(2) = 0$
- $f(1) = -4$
- $f(0) = -2$
- $f'(-1) = f'(1) = 0$
- if $x < -1$ then $f'(x) > 0$
- if $x > 1$ then $f'(x) > 0$

a. Draw a neat sketch graph of f

b. For which value(s) of x is f strictly decreasing and increasing?

c. Use your graph to determine the x-coordinate of the point of inflection of f .

d. For which value(s) of x is f concave up?

e. For which value(s) of x is $f''(x) < 0$.

Kwv 2

Given: $f(x) = ax^3 + bx^2 + cx + d$.

Draw a possible sketch of $y = f'(x)$ if a, b and c are all negative real numbers.

Kwv 3

A cubic function f has the following properties:

- $f\left(\frac{1}{2}\right) = f(3) = f(-1) = 0$
- $f'(x) = f'\left(-\frac{1}{3}\right) = 0$
- f decreases for x $\in [-\frac{1}{3}; 2]$ only.

Draw a possible sketch graph of f, clearly indicating the *x*-coordinates of the turning points and ALL the x-intercepts.

Kwv 4

Given: $f(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q (2: 10) are the turning points of f. the graph of f passes through the origin.

a) Show that $a = \frac{3}{2}$ and $b = 6$
b) Calculate the average gradient of f between P and Q, if it is given that $x = -1$ at P.
c) Show that the concavity of f changes at $x = \frac{1}{2}$.
d) Explain the significance of the change in c. with respect to f
e) Determine the value of x , given $x < 0$ at which the tangent to f is parallel to g

E. APLICATION OF CALCULUS

- **TO SHAPE**

Note:

- Perimeter: is the sum of all sides

Rectangle:………………. **Circle:……………….**

- Area: is the product of two sides

Rectangle:………………. **Circle:……………….**

- Prism Volume : is the area base times height

Rectangle:……………… **Cylinder:……………….**

- Pyramid Volume: is one-third area of base times height

Rectangular pyramid:………. **Cone**:…………………..

- Surface area: is the sum of all faces

Rectangle:…………………… **Cylinder**:………………

- Diameter: is the line passing through the centre from circumference

Diameter: ………………… **Radius:………………..**

NB: you must be in the position to imagine the shape if it is not given

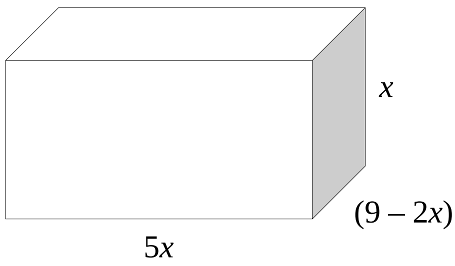
Key: To Application of Calculus

Maximum/ minimum/largest/smallest/at least/ at most and fixed etc. :

- Derive
- Let derivative be zero
- Solve
- Substitute into the original equation if necessary

Kwv 1

A rectangular box has a length of $5x$ units, breadth of $(9-2x)$ units and its height of x units.



a. Show that the volume (V) of the box is given by $V = 45x^2 - 10x^3$.

b. Determine the value of x for which the box will have maximum volume.

c. Hence, calculate the maximum volume.

d. Calculate the total surface area.

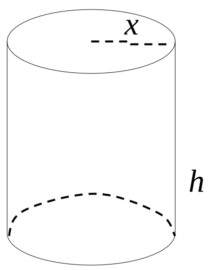
e. Determine the value of x for which the box will have maximum surface area.

f. Hence, calculate the maximum surface area.

Kwv 2

A container shaped in the form of a cylinder with **no top** has a volume of 340 ml.

It has a radius of x cm and a height of h cm. Note: 1 ml = 1 cm^3



a. Write down the height (h) in terms of x.

b. Show that the surface area (S) of the cylinder with no top is given by $S = \pi x^2 + \frac{680}{x}$.

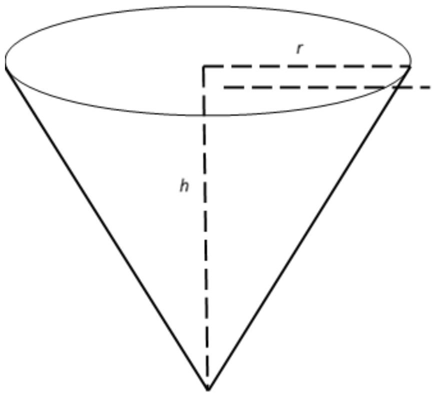
c. Calculate the value of x for which the surface area of the cylinder will be a minimum.

Kwv 3

A water tank in the shape of a right circular cone has a height of h cm. The top rim of the tank is a circle with radius of r cm. The ratio of the height to the radius is 5:2. Water is being pumped into the tank at a constant rate.

Surface Area of Cone = $\pi r^2 + \pi rs$

Volume of Cone $= \frac{1}{3}\pi r^2 h$

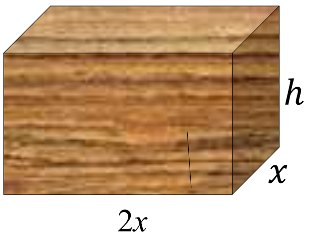


Determine the rate of change of the volume of water flowing into the tank when the depth is 5 cm.

- **COST**

Kwv 1

A crate used on fruit farms in the Ping River valley is in the form of a rectangular prism which is open on top. It has a volume of 1 cubic metre. The length and the breadth of its base is $2x$, and x metres respectively. The height is h metres. The material used to manufacture the base of this container costs R200 per square metre and for the sides, R120 per square metre



a. Express h in terms of x

b. Show that the cost, C, of the material is given by: $C(x) = 400x^2 + 360x - 1$

c. Calculate the value of x for which the cost of the material will be a minimum.

d. Hence, calculate the minimum cost of the material.

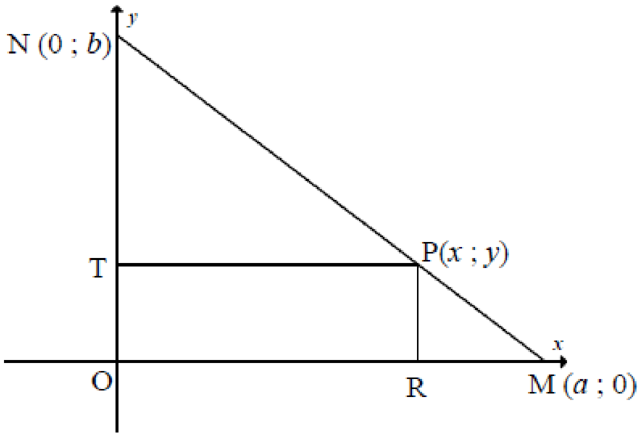
➢ TO GRAPH

You must recall the following:

- Distance/ length formula:……………………………
- Pythagoras formula:…………………………………
- Gradient formula:……………………………………
- Midpoint formula:……………………………………
- Line equation:………………………………………..

Kwv 1

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land PTOR to his daughter, giving her the freedom to choose P anywhere along the boundary MN. Let OM = a, ON = b and P(x ; y) be any point on MN.



a. Calculate the gradient of MN

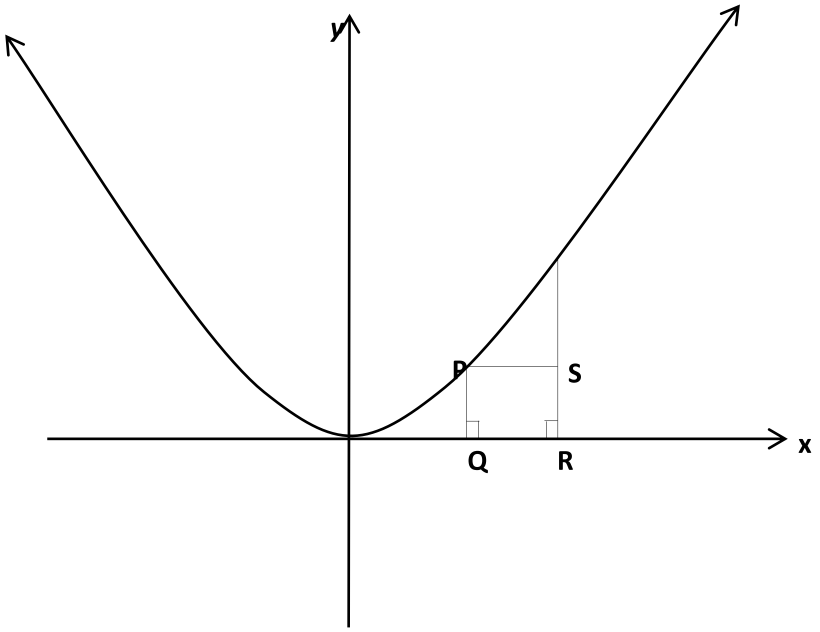
b. Determine an equation of MN in terms of a and b.

c. Calculate the midpoint of MN

d. Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN.

Kwv 2

The rectangle PQRS is drawn as shown in the sketch, with P a point on the curve $y = x^2$ and SR the line $x = 6$.



a. Write down the coordinates of Q, P and R

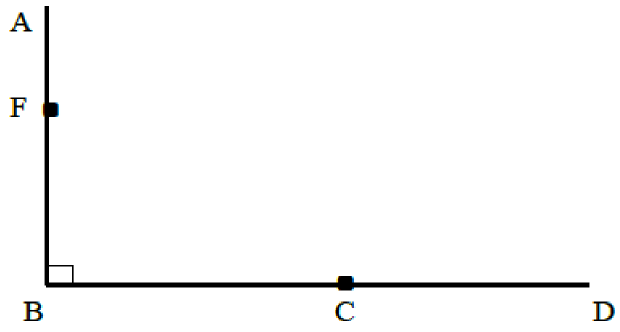
b. Express the length, QR, and breadth, SR, of the rectangle in terms of x.

c. Show that the area of the rectangle can be given as $A = -x^3 + 6x^2$.

d. Hence, calculate the area of the largest rectangle PQRS which can be drawn.

Kwv 3

Two cyclists start to cycle at the same time. One starts at point B and is heading due north to point A, whilst the other starts at point D and is heading due west to point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



a. Determine the distance between F and C in terms of t.

b. After how long will the two cyclists be closest to each other?

c. What will the distance between the cyclists be at the time determined in b?

➢ TO RATE

Rate:

- derive
- substitute with given value

At rest/ stationary/initial:

- time is zero
- velocity is zero

Flow of water:

- rate is flow in minus flow out

Rate of maximum:

- is the second derivative

Velocity/ speed:

- is the derivative of distance or displacement

Acceleration:

- is the derivative of velocity or second derivative of distance

Kwv 1

A stone is thrown vertically upward and its height (in metres) above the ground at (in seconds) is given by $h(x) = 35 + 5t - 5t^2 + 3t^3$

a. Find its initial height above the ground.

b. Find the initial speed with which it was thrown.

c. Determine the rate of change at $t = 35$.

d. Calculate the time at which the rate of change will be minimum.

Kwv 2

A tourist travels in a car over a mountainous pass during his trip. The height above sea level of the car, after t minutes, is given as $S(t) = 5t^3 - 65t^2 + 200t + 100$ meters . The journey lasts 8 minutes.

a. How high is the car above sea level when it starts its journey on the mountainous pass?

b. Calculate the car's rate of change of height above sea level with respect to time, 4 minutes after starting the journey on the mountainous pass.

c. Interpret your answer to QUESTION. b.

d. How many minutes after the journey has started will the rate of change of height with respect to time be a minimum?

Kwv 3

Round off your answer to the nearest whole number

A drink dispenser is able to fill up a 340 ml cup at a rate of x ml/s. if the rate increases to (x+2) ml/s, the time taken to fill up the cup will be reduced by 3 seconds.

Determine the original time taken to fill the cup

MERCY!!!!!

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